

# EIM 2025

## The Ecological Intelligence Module

### As Illustrated by the Cosmic Crossword Puzzle

### A Users Manual

Here is a very preliminary and incomplete draft of the EIM. When it is fully developed, I plan on using it as a template for a process control system for all TFT systems.

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### **Introduction:**

This Ecological Intelligence Module (EIM) is the initial illustration of how Ecological Intelligence can be applied to our personal experiences relative to the questions we ask and the decisions we make. It is based on the assumption that we think by creating and using language. We recognize that this includes our natural languages, which can be spoken or written, but it also includes additional forms of linguistic expression such as sculpture, art, dance, music, gestures, and similar modes of human interaction.

The EIM starts with the construction of one or more symbolic languages that can represent, with arbitrary accuracy, all of these ways we do think. Arbitrary accuracy in this sense requires that we do the best we can when we make these representations, and that we provide an estimate as to how much confidence we have in these representations.

For the initial illustration of EIM we will use the language structure from Ododu, a created language constructed from fundamental assumptions as to the nature of the universe and ourselves. The rationale for the construction of this structure is summarized in the Ododu section of this document.

To play the Cosmic Crossword Puzzle, or to use it in the Ecological Intelligence Module, you pick concepts and ideas from your experience and plug them into the Ododu structure. The goal is to find words that can be organized into coherent sentences and paragraphs which can assist us in making decisions that can help us find happiness, comfort, and satisfaction in our personal lives. I will present the choices I have made and what I have been able to do with these choices.

It is important to note here that this is a personal exercise that each of us can use to make decisions. The first step is to identify a particular dynamic situation in which we need to make decisions. Then try to fully describe this situation using the language structure that we have chosen, Ododu in this illustration. This description must include language that describes what has happened in the past and is happening now in the present. The language must also describe what can happen in the future, the possible futures that will be influenced or determined by the decisions that we will make.

Once this is done certain patterns can be extracted from the structure that describe the possibilities relative to the entire structure. They will represent possible decisions that can be made based on the information and organization of the language structure.

There are four possibilities as to how this analysis can be implemented. First, the operator or creator of the structure can simply select one of the possibilities based on their own intuition, whatever they feel is the best choice. Second, a statistical procedure can be developed and used based on any value structures which we may have included in the language description. This can be a simple random or Monte Carlo choice procedure or it may be constrained by other “relevant” information contained within the structure. This could be a function of some observable variable in organismal or systemic behavior in the system itself. Third,

it can be a combination of both of the above. Fourth, it could be implemented using a completely random number generator. A final wrinkle here would be to construct the random number generator within the system itself (and see if it continues to be completely random over time).

While the example presented here is based on philosophical ideas pertaining to the foundations of language, science, and mathematics, it can just as easily be applied to how your septic tank works or how to sustainably raise fish. As such it has evolved in concert with the development of the TimberFish Technologies and we plan to include it as a process control system for all TimberFish systems. Whether it be a 20 gallon table top model or a million pound per year aquaculture system.

This will be one way that people can learn to think ecologically. The EIM unit that will be included with the TT systems will be much simpler to understand and use than the presentation in this document, which describes the why and how of its development. Thus using it on a table top model fish tank, for example, will lead to an understanding of ecological decision making that will spread to other areas of peoples lives.

Thus Ecological Intelligence is local and personal. It incorporates what you think, and what the other living entities that live in your environment think. This is in stark contrast to Artificial Intelligence which only looks at the stochastic relationships among large compilations of data and does not consider the nature of the living sources of such data.

The development of the EIM has followed the same sort of evolutionary procedure that it embodies. Thus we use the achievements of history to guide us in the creation and use of EIM. In particular this will comprise; the creation of number, arithmetic, and geometry; the idea of debt and currency from economic and political systems; the idea of entropy from thermodynamics and information theory, and the use of quaternions in quantum mechanics. All grist for the mill that comprises the languages of Ecological Intelligence.

### **Ododu:**

This constructed language starts with the assumption that the most fundamental concepts that we can use to describe the nature of ourselves and the universe comes from the notion of relation. We recognize four such concepts, self relation, linear relation, relational relation, and interrelational relation. Here are four signs that represent an initial symbolic formalism of these concepts.

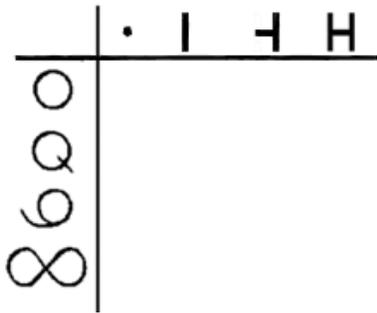


These are reflected to generate a second symbolic formalism.

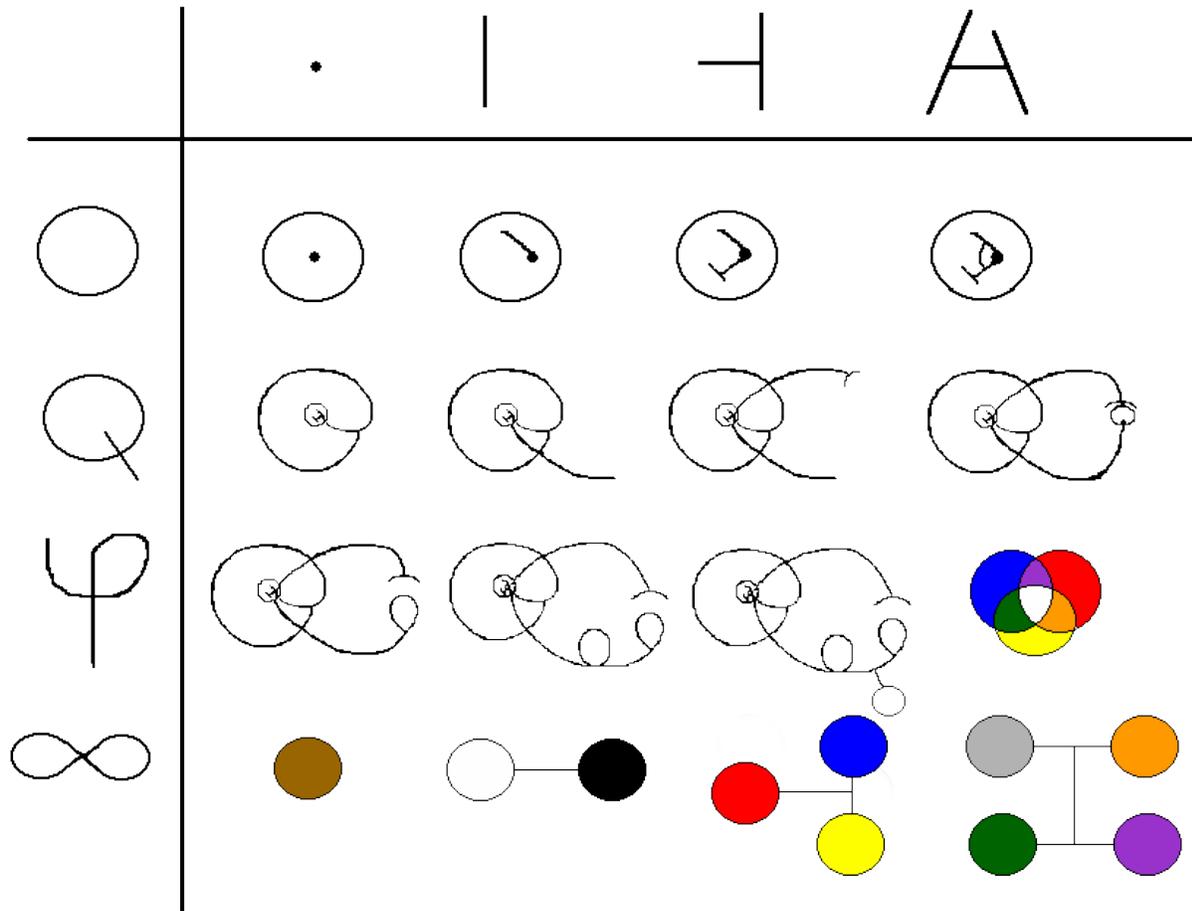


These are described as a boundary or distinction, a crossing of the boundary, a mark as to where you are after crossing (inside or outside), and a connection of all these concepts.

We organize these symbols as follows.



And use them to derive additional symbols that represent archetypal concepts relative to our understanding of ourselves and the universe. Here are symbolic diagrams of these additional archetypal concepts.



An example of how these symbols can be constructed can be found in the “Derivation of Archetypal Meaning” document in [link](#). This leads to the concept of a Cosmic Crossword Puzzle in which each of us tries to assign words to each of these symbols that describe our view of ourselves and the universe within which we live.

For convenience of use let each of these symbols be represented by a letter in the Roman alphabet, as follows.

	U	I	E	A
O	D	P	R	B
Q	C	L	T	K
Y	S	G	F	X
H	N	M	W	Z

We can now assign meanings to each of these letters and use them to create a language, our personal solution to the Cosmic Crossword Puzzle. For example, my current solution is to assign meaning to the letters as follows;

	Noun	Verb	Modifier	Relational
Mind	Consciousness	Desire	Emotion	Memory
Matter	Body	Action	Sensation	Creation
Word	Symbol	Definition	Image	Idea
World	Thing	Interaction	Property	Connection

Thus the top row initial relational concepts can be seen as representing a structure we all recognize as relating to language. The left hand column represents areas of our experience that the archetypal concepts describe.

Thus the 16 archetypal noun concepts can be expressed as a 4 by 4 array;

Consciousness	Desire	Emotion	Memory
Body	Action	Sensation	Creation
Symbol	Definition	Image	Idea
Thing	Interaction	Property	Connection

This language has a grammar, a provision that distinguishes between words that represent ideas and those which represent things or phenomena we experience as real, and a procedure for generating additional words with increased specificity in each of the 16 major categories. This includes how combinations of words in separate categories can be interrelated to create new meanings and hence more detailed understandings.

We my personal choice of archetypal meanings this procedure generates the basic concepts that generate the four fields of integer, rational, real, and complex numbers. It also generates the fundamental systems of mathematics including set theory, geometry, algebra, groups, topology, probability, etc. Detailed descriptions and additional information on Ododu is in Appendix A.

### **Probabilities:**

The basic motivation for the development of EIM is how to make better decisions. Thus we collect information from the present and the past and use it to make predictions as to what will happen in the future. Since in general we do not know exactly what will happen in the future we create the idea of probability. Thus the first question is what is a probability and how can we express it with language. For insight on this question we turn to Richard Cox and his book on “The Algebra of Probable Inference” (with particular note of pgs. 12, 17, 19, 40-42.)

Following his lead we will define a probability in terms of inferences based on hypotheses. Since we are using the English language for this discussion we will consider that an inference will be a sentence, and a hypothesis will be one or more paragraphs comprised of sentences. Generally the sentences used in the hypothesis will not be the same as the sentences used to describe inferences based on that hypothesis.

Denote an inference as  $i$ , and a hypothesis as  $h$ . Then  $(i|h)$  represents a probability as a measure of assent of  $i$  on  $h$ , that is, a measure of the likelihood of  $i$  given  $h$ . To be useful this requires that for a given hypothesis  $h$  there exists an exhaustive set of inferences that can be defined on  $h$ .

Given a language array such as described for Ododu above we represent the hypothesis as comprising the array;

Consciousness	Desire	Emotion	Memory
Body	Action	Sensation	Creation
Symbol	Definition	Image	Idea
Thing	Interaction	Property	Connection

Each word can be used as a noun, verb, modifier, or relational.

Define a sentence as a collection of words comprising at least one of a noun, verb, modifier, and relational. Thus in the array above there are one or more sentences associated with each word in the array. To describe our hypothesis  $h$ , we require that none of the sentences used to define the array are contradictory with each other.

We now require that for a sentence to be an inference it should contain one word from each row and matrix in the array.

(Thus any inference would be considered as an inference based on the array of all of the archetypal meanings in the Ododu language. For example, if the word consciousness is to be used in an inference it must be understood in terms of all of the other concepts in the conceptual framework that comprises the language of Ododu.

This would also hold for much more narrowly defined subjects as long as the hypothesis represents a complete and exhaustive description of the subject.

For a probability  $(i|h)$  to be measurable it needs to be associated with a number. For example  $a(i|h)$ , where  $a$  is a real number, represents a probability that has been, or can be, measured with a result of  $a$ . This will also hold for functions of  $(i|h)$  such as  $(i|h)(i|h)$  or  $\ln(i|h)$ . This seems straightforward for the integers, rational, and real numbers but raises a question relative to the complex numbers. This involves the issue of what are negative numbers.

If we look at human history it is clear that our understanding and use of numbers and mathematics has changed and evolved over time. Counting numbers, fractions, and ratios, in conjunction with arithmetic and geometry, were influenced by economic systems and currencies. The concept of a debt, a measurable future obligation, provided a tangible reality emerging from human interaction that could be represented with negative numbers. This converted arithmetic systems into algebras and generated the idea of complex numbers. But it also illustrates that

positive numbers are adequate to describe the present and the past, but that negative numbers are essential if we want to describe the future.

What this means relative to our language array is that if we want to use probabilities to help us make decisions in the future, then we need both positive and negative words, words that connote future as well as past or present, to construct our inferences.

With this as context lets look at the quaternions.

Note here that evolution looks at the past and builds on that for the future.

### **Quaternions:**

A quaternion is a number with a form of;

$$a1 + bv + cj + dk$$

Where a, b, c, and d are real numbers, 1 is the unitary concept of one, and v, j, and k are non equivalent imaginary numbers each equal to the square root of minus one. In addition to being a well defined algebra comprising addition, subtraction, multiplication, and division, these numbers must also satisfy the following conditions. Let \* represent multiplication, then

$$v * v = -1, \quad j * j = -1, \quad k * k = -1$$

$$v * j = -j * v = k$$

$$j * k = -k * j = v$$

$$k * v = -v * k = j$$

and  $v * j * k = -1$

but v, j, and k are not equal to each other. This notation does not provide any clarity

as to how the squares of  $v$ ,  $j$ , and  $k$  can all be equal to minus one when  $v$ ,  $j$ , and  $k$  are not equal to each other. This can be resolved by expanding the definition of the quaternion bases to four by four matrices using only one, minus one, and zero. This matrix notation provides an illustration of how the quaternion basis elements of  $v$ ,  $j$ , and  $k$  can all equal the square root of minus one without being equal to each other.

In Appendix A, 8 known quaternions are expressed with 4 by 4 matrices with only 0, 1, and -1 used as digits. The first quaternion example, Q1.1, comprises the following matrices.

Identity matrix 1

$$\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}$$

$v1$

$$\begin{array}{cccc} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{array}$$

$j1$

$$\begin{array}{cccc} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{array}$$

$k1$

$$\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{array}$$

The last three matrices all square to the negative of the identity matrix, -1.

$$\begin{array}{cccc} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array}$$

The first matrix is the identity matrix and it is the same for all 8 quaternions. The remaining three matrices,  $v$ ,  $j$ , and  $k$ , represent the rotating basis elements that all square to minus one. There are 12 total such matrices, 6 pairs with each matrix in a pair being the negative of the other.

See Appendix A for a listing of all 8 quaternions in 4 by 4 matrix notation.

I will be generating a series of these mappings with attempts at determining their meaning in the near future.

### **The Maximum Entropy Principle (MEP):**

I have talked before about how the MEP was derived and how it is now usually presented.

The basic equation is;

$$S = [-p(j) \ln p(j) + \lambda^*(p(j) - 1) + \sum_u (\lambda_u p(j)F(u,j) - \langle F_u \rangle)]$$

Where  $S$  is the constrained entropy,  $\lambda$  represents a Lagrange multiplier, and there are  $n$  different  $p(j)$  probabilities and  $m$  different  $\langle F_u \rangle$  expected value functions.

See Appendix D for a more detailed explanation of the Maximum Entropy Principle.

To show how the Maximum Entropy Principle can be applied with respect to Ododu consider a system  $X$ , that comprises a comprehensive linguistic description of some aspect of our experience and/or understanding of ourselves or the universe. We can represent this description with the Ododu 4 x 4 array The array

should include everything that we think we know about the system expressed in a consistent, comprehensive, and non contradictory manner.

If we map this array into the quaternion structure where the quaternion is expressed with only with one, minus one, and zero as described above, then we can define a hypothesis as comprising the identity matrix of the quaternion.

Then define inference statements based on this hypothesis as being the mappings of the 12 rotating quaternions basis elements described above onto X, the 4 by 4 matrix containing the 16 archetypal concepts. The question now becomes how to define the entropy term as  $(i|h)\ln(i|h)$  which represents the entropy term of  $-p(j) \ln p(j)$  as used above.

This involves the concept of logarithm which is considered to be the inverse of exponentiation. Thus if

$$A^n = B$$

$$\text{Then } \log_A(B) = n$$

A is defined as the base of the logarithmic system and n is defined as the power that must be used on A to get B.

For example  $10^2 = 100$ , can be expressed as

$$\log_{10}100 = 2$$

The choice of what to use as the base for a logarithmic system is arbitrary. For example the number e is commonly used as the base for the “natural logarithm” where e is 2.718281828.... Thus  $\log_e(B) = n$  represents  $e^n = B$ .

This is usually written as  $\ln(B) = n$ .

These definitions of logarithms use real numbers as bases and there are logarithmic expressions for matrices which also use real numbers as bases. This does not translate well to a system that is based solely on linguistic descriptions not involving or requiring real numbers. However, the structures of mathematics that are based on linguistic statements that define the creation and use of numbers in general, and real numbers in particular, are a useful guide as to how to structure a linguistic system that can evolve to generating mathematics, including the number systems.

To do this within the Ododu 4 by 4 array representation we could define the identity matrix of a quaternion map onto the linguistic array as being the base for a “logarithmic” function within that array. This would then represent the hypothesis,  $h$ , which we have constructed within that array. Then we could define the inferences,  $i$ , based on the hypothesis as being the mappings of the 12 other rotating quaternion matrices which square to the negative version of the identity matrix as the inferences based on the hypothesis formulated for the system described by  $X$ .

The problem is that logarithmic functions are defined with single valued real numbers for bases. Complex numbers in matrix notation are not single valued and cannot be ordered. Instead they represent collections of non orientable patterns, and that these collections repeat as non orderable patterns in orderable sequences.

Using the notation of  $(i|h)$  to denote these 12 matrices define

$$\log_h(B) = C$$

where  $B$  and  $C$  are both inferences defined on  $X$  relative to  $h$  and

$$h^B = C$$

Thus the hypothesis  $h$  will be raised to some “power” or “exponential” such that

$$\log_h(B) = C$$

$$A^n = B$$

$$\text{Then } \log_A(B) = n$$

For example  $10^2 = 100$ , can be expressed as

$$\log_{10}100 = 2$$

$$100/10 = 2$$

$\text{Log}_h$  as being a logarithmic function defined on the Ododu array where the identity matrix map onto the array comprises the base for the linguistic description  $X$  of that array.

If we consider that  $\ln(i|h)$  is the negative or inverse of  $(i|h)$  then  $(i|h)\ln(i|h)$  will be equal to the unitary matrix, 1, while  $(i|h)(i|h)$  will be equal to the negative unitary matrix, -1.

(Note: In 1 u Matrix Sentences with 12 one or minus one 3-10-25

Rows 111 through 114 show that not all matrices with two plus and two minus ones, when multiplied by their inverses, equal the plus or minus unitary matrix. Thus the maximum entropy procedure cannot be used for any inferences which have such a structure. Hence the MEP can only be applied to the eight known quaternions described in this document.)

Consider the Q1.1 quaternion described on pages 9 and 10.

Let  $v_1$  be the first inference  $(i|h)$ .

$$\begin{array}{cccc} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{array}$$

Then the inverse of  $v_1$  will be  $-v_1$ ;

$$\begin{array}{cccc} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{array}$$

$v_1 * v_1 = -1$ , the negative unitary matrix.

$v_1 * -v_1 = 1$ , the positive unitary matrix.

$\ln v_1 = -v_1$ , So

$$v_1 \ln v_1 = v_1 * \ln v_1 = 1$$

By the same reasoning

$$j_1 \ln j_1 = 1, \text{ and}$$

$$k1lnk1 = 1$$

This will hold for all eight quaternions described in the 8 known quaternions.

These eight quaternions that can be constructed from these statement mappings comprise an exhaustive set of inferences defined on X. Thus these eight quaternion “paragraphs”, comprising four basis matrices, “sentences” is a Maximum Entropy description of the 16 archetypal Ododu hypothesis definitions.

If you can define these archetypal concepts, or some other representation of a problem describable in a similar manner, then you could do an actual MEP calculation.

Denote  $p(u)$  as the probability that  $Q(u)$  is the best inference we can make on this hypothesis,  $u = (1 \text{ to } 8)$ . Then you could “do the math”. (Seems very complicated at this point, but could be done.)

When we cannot, or don’t want to, define functions for the relational terms that are representable with numbers we can still make balanced (aesthetic or artistic) judgements about our description. These would not be calculated in the way that the MEP can be calculated given precise inferences based on a hypothesis, because we do not have a precise understanding of the signs in terms of inferences and hypotheses. However, the Maximum Entropy solution can be sensed by a living conscious organism in a manner that is comparable to how entropy exists in the language of that organism. This goes back to how Ododu represents how our language in universal terms. Thus we combine the linguistic representation with the quaternion basis elements.

### **EIM 2025:**

So here is an example of how this could be applied to the first quaternion, Q1.1. Note that negative words are about the future whereas positive words are about the past as understood in the present. Here is the unitary basis element. This will be the same for all eight quaternion paragraphs.

$$\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}$$

Mapping the one unitary matrix into the Archetypal Concept nouns gives

Consciousness	0	0	0
0	Action	0	0
0	0	Image	0
0	0	0	Connection

This can be viewed as a sentence where consciousness is a noun, action is a verb, image is a modifier for action, and connection is a relational term for connectivity or covariance. Thus: Consciousness performs an image action of covariance. Or Consciousness images covariance. Consciousness images connectivity. I can interpret this to mean;

Consciousness does an image action of connection. Or

I do illustration of covariance. (my version) or

I do art of connectivity. (Lynn's version).

In this interpretation "Consciousness images covariance" can be the unitary matrix (sentence) for eight different four sentence paragraphs (quaternions) defined on the 16 archetypal nouns of Ododu.

Then

0	1	0	0
-1	0	0	0
0	0	0	-1
0	0	1	0

Maps into

0	want	0	0
Future body	0	0	0
0	0	0	Past idea
0	0	Future characterization	0

Which I interpret to mean;

My future self will want a future characterization of past ideas.

Then

0	0	0	-1
0	0	1	0
0	-1	0	0
1	0	0	0

Maps into

0	0	0	Future beliefs
0	0	Sensation	0
0	Future definitions	0	0
Systems	0	0	0

Which I interpret to mean;

Systems will define sensations of future beliefs.

Then

0	0	1	0
0	0	0	1
-1	0	0	0
0	-1	0	0

Maps into

0	0	Emotion	0
0	0	0	Creation
Future signs	0	0	0
0	Future forces	0	0

Which I interpret to mean;

Future signs and future interactions will result from how we feel now about our current creations.

This describes the linguistic first paragraph quaternion of the eight paragraph quaternions that comprise the Golidlocks Quaternion Formulation of the Maximum Entropy Principle.

Need to describe how this can be extended to many different kinds of applications and reference the TFT process as an example of what we will be doing.

### **Appendices:**

Appendix A Ododu

Appendix B Probabilities

Appendix C Quaternions

Appendix D Maximum Entropy Principle

Appendix E Gell-Mann Matrices

### **Appendix A Ododu**

(links to Ododu, RSP, Philosophers Dream, )

### **Appendix B Probabilities**

Reference Richard Cox, Algebra of Probable Inference, pgs. 12, 17, 19, 40-42.)

## Appendix C see 1 u 8 Known quaternions 3-9-25

		v					j					k			
Q1.1	v1	0	-1	0	0	j1	0	0	0	1	k1	0	0	-1	0
		1	0	0	0		0	0	1	0		0	0	0	1
		0	0	0	-1		0	-1	0	0		1	0	0	0
		0	0	1	0		-1	0	0	0		0	-1	0	0
Q1.2	-v1	0	1	0	0	-j1	0	0	0	-1	k1	0	0	-1	0
		-1	0	0	0		0	0	-1	0		0	0	0	1
		0	0	0	1		0	1	0	0		1	0	0	0
		0	0	-1	0		1	0	0	0		0	-1	0	0
Q1.3	v1	0	-1	0	0	-j1	0	0	0	-1	-k	0	0	1	0
		1	0	0	0		0	0	-1	0		0	0	0	-1
		0	0	0	-1		0	1	0	0		-1	0	0	0
		0	0	1	0		1	0	0	0		0	1	0	0
Q1.4	-v1	0	1	0	0	j1	0	0	0	1	-k	0	0	1	0
		-1	0	0	0		0	0	1	0		0	0	0	-1
		0	0	0	1		0	-1	0	0		-1	0	0	0
		0	0	-1	0		-1	0	0	0		0	1	0	0
Q2.1	v2	0	1	0	0	j2	0	0	0	-1	k2	0	0	1	0
		-1	0	0	0		0	0	1	0		0	0	0	1
		0	0	0	-1		0	-1	0	0		-1	0	0	0
		0	0	1	0		1	0	0	0		0	-1	0	0
Q2.2	-v2	0	-1	0	0	-j2	0	0	0	1	k2	0	0	1	0
		1	0	0	0		0	0	-1	0		0	0	0	1
		0	0	0	1		0	1	0	0		-1	0	0	0
		0	0	-1	0		-1	0	0	0		0	-1	0	0

Q2.3	v2	0	1	0	0	-j2	0	0	0	1	-k2	0	0	-1	0
		-1	0	0	0		0	0	-1	0		0	0	0	-1
		0	0	0	-1		0	1	0	0		1	0	0	0
		0	0	1	0		-1	0	0	0		0	1	0	0
Q2.4	-v2	0	-1	0	0	j2	0	0	0	-1	-k2	0	0	-1	0
		1	0	0	0		0	0	1	0		0	0	0	-1
		0	0	0	1		0	-1	0	0		1	0	0	0
		0	0	-1	0		1	0	0	0		0	1	0	0

## Appendix D The Maximum Entropy Principle

Classically the Maximum Entropy Principle (MEP) is a generalized method of logical inference which is defined on a system in terms of inferences based on hypotheses.

The procedure is to pick a system that can be described in terms of a state variable, call it  $X$ , which can take on a series of numerical values. Thus  $X$  represents a hypothesis that the state variable exists, that it can be exhaustively described in terms of a fixed number of inferences defined on that hypothesis, and that these inferences can be associated with specific numerical values. Assign a probability to each of these possible values,  $p(j)$ , such that  $p(j)$  describes the probability that the  $j^{\text{th}}$  value will occur. This then becomes a number associated with the linguistic statement that the  $j^{\text{th}}$  inference will occur given the linguistic hypothesis presented by the state variable  $X$ .

(For example, consider a die, a cube with six faces and a number from one through six on each face with no repetitions. The  $p(j)$  represent the probabilities that when the die is thrown, the  $j^{\text{th}}$  face will be on top.)

If there is no additional information then the entropy  $S$  of the general situation is represented by;

$$S = - \sum_j p(j) \ln p(j)$$

Where there are  $n$  possible inferences based on the hypothesis of  $X$ . In the dice example  $n$  is equal to 6 in that there are six possible outcomes of throwing the die.

Usually this term is constrained or normalized by the condition that the sum of all  $n$  probabilities must equal one.

If there is additional information available such that one or more expected value functions,  $F(u,j)$ , can be defined on  $X$  then these provide additional constraints for a calculation of the best probability distribution for  $p(j)$ . Here  $u$  is an index for the number of such expected value functions that are available such that;

$$\langle F_u \rangle = \sum_j p(j)F(u,j)$$

Can be summed over all possible values of  $j$ , and  $\langle F_u \rangle$  is the expected value for the  $F(u,j)$  function.

Given  $m$  available expected value functions the MEP can be calculated using the method of Lagrange multipliers to determine the “best” probability distribution for  $p(j)$ , where best is usually interpreted in terms of maximizing our uncertainty for any situation or decision. The basic equation is;

$$S = [-p(j) \ln p(j) + \lambda^*(p(j) - 1) + \sum_u (\lambda_u p(j)F(u,j) - \langle F_u \rangle)]$$

Where  $S$  is the constrained entropy,  $\lambda$  represents a Lagrange multiplier, and there are  $n$  different  $p(j)$  probabilities and  $m$  different  $\langle F_u \rangle$  expected value functions. Setting  $S$  equal to zero and differentiating with respect to  $p(j)$  gives;

$$-1 - \ln p(j) + \lambda^* + (\sum_u \lambda_u F(u,j))$$

Setting  $\lambda_0 = \lambda^* - 1$  gives

$$P(j) = e \exp \lambda_0 + e \exp (\sum_u \lambda_u F(u,j))$$

An alternative way to view this situation is to use numerical techniques to iteratively select values for  $p(j)$  and  $\lambda_u$  such that  $S$  will be a maximum.

In the GMEP we will express the MEP in terms of a quaternion. Thus we will interpret the entropy term as representing a unitary 4 by 4 matrix, and consider three expected value functions as the three non commuting basis elements of a quaternion.

We now can express the expected value functions of the classical MEP as comprising a quaternion basis element and an informational “expected Value” statement as follows.

Consider a description of a state variable  $X$  that represents a Relational Symmetry, for example

•  $|$   $\dashv$   $H$  . Let the  $\bullet$  relation represent the entropy term, and the other three relational forms represent three expected value functions. Thus

$$\begin{aligned} \bullet & \text{ represents } \Sigma_j [-p(j) \ln p(j)] \\ | & \text{ represents } \langle F_1 \rangle - \Sigma_j [\lambda_1 p(j)F_1(|,j)] \\ \dashv & \text{ represents } \langle F_2 \rangle - \Sigma_j [\lambda_2 p(j)F_2(\dashv,j)] \\ H & \text{ represents } \langle F_3 \rangle - \Sigma_j [\lambda_3 p(j)F_3(H,j)] \end{aligned}$$

Then require that  $\Sigma_j [p(j)] = 1$

If we could define functions for the  $|$  ,  $\dashv$  ,  $H$  relations that are expressible with numbers we could solve this equation by choosing values for  $\lambda_1$ ,  $\lambda_2$ ,  $\lambda_3$ , and the  $p(j)$  such that the sum of the four  $\bullet$   $|$   $\dashv$   $H$  constraints is a maximum. This would balance our Maximum Entropy equation. The  $p(j)$  would then be the probability that any given sign in this symbolic formalism will be relationally symmetrical with the other three signs.

### Appendix E? Gell-Mann matrices

These have structures that can be related to the use of quaternions and will be a project for the future.

$$\lambda_1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\lambda_4 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\lambda_7 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{bmatrix}$$

$$\lambda_2 = \begin{bmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\lambda_5 = \begin{bmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{bmatrix}$$

$$\lambda_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\lambda_6 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\lambda_8 = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix},$$